CALCULATION OF TEMPERATURE FIELD IN PLASMATRON ELECTRONS

B. A. Uryukov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 5, pp. 84-91, 1967

The problems of calculation of temperature fields in the electrodes of a plasmatron (electric-arc heater) differ from the usual heat-conduction problems in that in a plasmatron there is not only convective heat transfer between the electrode wall and the working medium and between the wall and the cooling medium, but also an intense delivery of heat to the electrode in a sharply delimited region of the electrode surface–the arc anchor spot.

In [1] the source-and-sink method was used to consider several similar problems in application to the heat calculation of electrodes for welding. In [2–5] the temperature fields from a round stationary source on the surface of a semi-infinite cooled plate and from the spot of an arc moving at high speed between two coaxial cylinders (electrodes) were investigated.

We consider a plasmatron electrode in the form of a hollow cylinder, one surface of which is traversed by the arc spot and the other surface is cooled. We will assume that the spot moves in a closed path in one cross section of the electrode. The electrode could have other configurations, such as a disk, with the spot moving over its surface in a circular path.

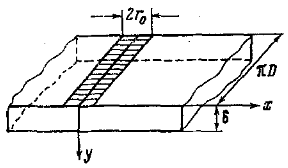

Fig. 1

We assume that the heat flux from the working medium to the electrode wall outside the spot is due entirely to convection, and in the spot the heat flux Q from the electrode region of the arc is added to it. We consider a round spot. The specific heat flux q_0 (per unit area of the spot) from the electrode region of the arc is assumed to be constant and distributed uniformly over the spot area

$$q_0 = Q / \pi r_0^2 . \tag{0.1}$$

Here r_0 is the spot radius. The temperature of the working medium T_{01}, the cooling medium T_{02}, the heat transfer coefficients of the working medium α_1 and cooling medium α_2, the thermal conductivity λ and thermal diffusivity a of the electrode material are assumed to be constant. The electrodes are of infinite length. We consider the case of steady-state heating.

1. Cylindrical electrode with spot moving at high speed (plane approximation). If the spot moves rapidly enough over the surface of a cylindrical electrode we can assume that the spot degenerates into a ring of width $2r_0$–the "smeared" spot. The simplest solution of the problem of the temperature field in the electrode is found by the "plane-approximation" scheme, where the cylindrical electrode is imagined to be cut along a generatrix and opened up to form a plate of width πD and thickness δ (Fig. 1), where D is some mean diameter of the initial cylinder. The problem now becomes a plane one. The heat-conduction equation is

$$\partial^2 T'/\partial x^2 + \partial^2 T'/\partial y^2 = 0 \quad (T' = T - T^*). \tag{1.1}$$

Here T is the temperature of a given point on the plate; T^* is the temperature of a point on the plate at an infinite distance from a spot with the same coordinate y as the given point; this temperature depends only on the convective heat transfer between the electrode and the working and cooling media.

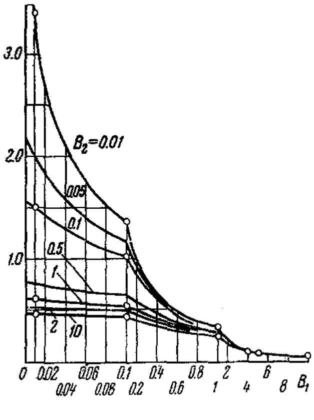

Fig. 2

The specific heat flux from the electrode region of the arc in the smeared spot is

$$q = Q/2\pi D r_0 . \tag{1.2}$$

The boundary conditions of the problem are

$$\lambda \partial T' / \partial y = -qH(r_0^2 - x^2) + \alpha_1 T', \quad y = 0;$$

$$\lambda \partial T' / \partial y = -\alpha_2 T', \quad y = \delta;$$

$$\partial T' / \partial x = 0, \quad x = 0. \tag{1.3}$$

Function H is defined in the following way:

$$H(z) = 1 \text{ when } z \geqslant 0,$$

$$H(z) = 0 \text{ when } z < 0. \tag{1.4}$$

We introduce the dimensionless parameters

$$\xi = \frac{x}{\delta}, \quad \eta = \frac{y}{\delta}, \quad B = \frac{\alpha\delta}{\lambda},$$

$$\tau = \frac{\pi T' \lambda \delta}{Q}, \quad \nu = \frac{D}{\delta}, \quad \rho = \frac{r_0}{\delta}.$$

Here B is the Biot number. The heat-conduction equation takes the form

$$\partial^2\tau/\partial\xi^2 + \partial^2\tau/\partial\eta^2 = 0 . \tag{1.5}$$

The boundary conditions are

$$\frac{\partial\tau}{\partial\eta} = -\frac{H(\rho^2 - \xi^2)}{2\rho\nu} + B_1\tau, \quad \eta = 0;$$

$$\frac{\partial\tau}{\partial\eta} = -B_2\tau, \quad \eta = 1; \quad \frac{\partial\tau}{\partial\xi} = 0, \quad \xi = 0 . \tag{1.6}$$

The temperature T^* is given by the formula

$$T^* = \frac{B_1(1 + B_2)T_{01} + B_2T_{02} - B_1B_2(T_{01} - T_{02})\eta}{B_1 + B_2 + B_1B_2} . \tag{1.7}$$

The problem is solved by the Fourier method by representing the even function $f(z)$ in the form of a Fourier integral [6],

$$f(z)=\frac{1}{\pi}\int_{-\infty}^{\infty}\left[\int_{0}^{\infty}f(t)\cos pt\,dt\right]\cos pz\,dp\,, \tag{1.8}$$

so that

$$H(\rho^2-\xi^2)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin p\rho}{p}\cos p\xi dp\,\cdot \tag{1.9}$$

Satisfying the boundary conditions (1.6) and using the evenness of the integrand in the expression for τ we finally obtain

$$\nu\tau(\xi,\ \eta)=\frac{1}{\pi\rho}\int_{0}^{\infty}\frac{\cos p\xi\sin p\rho}{p}\times$$
$$\times\frac{p\operatorname{ch}p(1-\eta)+B_2\operatorname{sh}p(1-\eta)}{(p^2+B_1B_2)\operatorname{sh}p+p(B_1+B_2)\operatorname{ch}p}dp\,. \tag{1.10}$$

Figure 2 shows characteristic relationships between $\nu\tau(0, 0)$ (value of $\nu\tau$ at center of spot) and the Biot numbers B_1 and B_2 for $\rho = 1$. It should be noted that if B_2 is sufficiently large $\nu\tau(0, 0)$ depends weakly

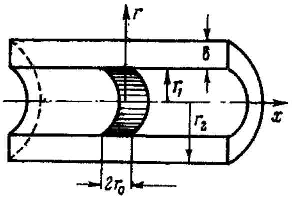

Fig. 3

on B_2. When B_1 is large $\nu\tau(0, 0)$ is practically independent of B_2 and depends weakly on B_1.

In the cavity of the cylindrical electrode of a plasmatron, where aerodynamic forces due to the moving working medium act on the arc, oscillations of the arc length occur and the distribution of the specific heat flux from the arc spot through the zone of oscillations of the arc, the length of which is l, can be regarded as almost normal

$$q=q_1\exp\ (-x^2/r_1^2). \tag{1.11}$$

The total heat flux brought to the spot is

$$Q=\pi\sqrt{\pi}\ Dq_1r_1\Phi\ (l/2r_1). \tag{1.12}$$

Here $\Phi(z)$ is a probability integral.

Since this heat flux is equal to the flux from an arc spot of radius r_0 when the specific heat flux $q = q_0(0.1)$, then

$$q_1=\frac{q_0r_0^2}{\sqrt{\pi}\,r_1D\Phi\,(l/2r_1)}\cdot \tag{1.13}$$

The solution of the problem of the temperature field of a cylindrical electrode in the plane approximation with normal distribution of the specific heat flux (1.11) leads to the relationship (for case $l/2r_1 \gg 1$, when $\Phi(l/2r_1) \approx 1$),

$$\nu\tau(\xi,\ \eta)=\frac{1}{\pi}\int_{0}^{\infty}\cos p\xi\exp\left(\frac{-p\rho^2}{4}\right)\times$$
$$\times\frac{p\operatorname{ch}p(1-\eta)+B_2\operatorname{sh}p(1-\eta)}{(p^2+B_1B_2)\operatorname{sh}p+p(B_1+B_2)\operatorname{ch}p}dp\,. \tag{1.14}$$

Here $\rho = r_1/\delta$. Calculations from (1.14) (points in Fig. 2) show that $\nu\tau(0, 0)$ in this case differs little from $\nu\tau(0, 0)$ when the specific heat flux is distributed uniformly over the spot area.

2. Cylindrical electrode with spot moving at high speed. The results obtained above are valid for thin-walled electrodes. If the ratio of the wall thickness to the electrode diameter cannot be regarded as small, the problem has to be solved by a more accurate method.

We consider a cylindrical electrode (Fig. 3). The inner surface of the electrode, of diameter $D = 2r_1$, receives a heat flux in a smeared annular spot of width $2r_0$. The electrode outside diameter $D_2 = 2r_2$. The x-axis is directed along the electrode axis and r is the radius of the given point. The wall thickness $\delta = r_2 - r_1$. We introduce the dimensionless parameters

$$\xi=\frac{x}{\delta}\,,\quad \eta=\frac{r}{\delta}\,,\quad B=\frac{\alpha\delta}{\lambda}\,,\quad \tau=\frac{\pi T''\lambda\delta}{Q}\,,$$
$$\nu_1=\frac{D_1}{\delta}\,,\ \nu_2=\frac{D_2}{\delta}\,,\ \rho=\frac{r_0}{\delta}\,,\quad R=\frac{r_2}{r_1}\,.$$

As a reference dimension we take the wall thickness, so that we can compare the solution with the plane-approximation solution.

The heat-conduction equation and boundary conditions take the form

$$\frac{\partial^2\tau}{\partial\xi^2}+\frac{1}{\eta}\frac{\partial}{\partial\eta}\eta\frac{\partial\tau}{\partial\eta}=0\,,$$
$$\frac{\partial\tau}{\partial\eta}=-\frac{1}{2\rho\nu_1}H(\rho^2-\xi^2)+B_1\tau,\qquad \eta=\frac{1}{R-1}\,, \tag{2.1}$$
$$\frac{\partial\tau}{\partial\eta}=-B_2\tau,\qquad \eta=\frac{R}{R-1}\,,$$
$$\frac{\partial\tau}{\partial\xi}=0,\qquad \xi=0\,. \tag{2.2}$$

The temperature T' is defined as in Section 1, and the temperature T* is given by the formula

$$T^*=\left[\frac{B_1}{R}T_{01}+B_2T_{02}-\frac{B_1B_2}{R-1}\times\right.$$
$$\left.\times\left(T_{01}\ln\eta\frac{R-1}{R}-T_{02}\ln\eta\,(R-1)\right)\right]\times$$
$$\times\left(\frac{B_1}{R}+B_2+\frac{B_1B_2}{R-1}\ln R\right)^{-1}. \tag{2.3}$$

Table 1

X	a	b	X	a	b	X	a	b
0	0	2.6124	17	1.0251	0.09756	33	0.5744	—0.5784
1	0.5457	1.9716	18	1.0073	0.03664	34	0.5385	—0.6029
2	0.7158	1.7073	19	0.9975	—0.02127	35	0.5021	—0.6256
3	0.8240	1.5057	20	0.9659	—0.07632	36	0.4652	—0.6456
4	0.9003	1.3369	21	0.9427	—0.1287	37	0.4280	—0.6656
5	0.9562	1.1896	22	0.9180	—0.1784	38	0.3903	—0.6830
6	0.9976	1.0576	23	0.8919	—0.2256	39	0.3522	—0.6987
7	1.0280	0.9376	24	0.8645	—0.2705	40	0.3138	—0.7128
8	1.0498	0.8271	25	0.8359	—0.3131	41	0.2752	—0.7251
9	1.0645	0.7246	26	0.8063	—0.3534	42	0.2363	—0.7358
10	1.0733	0.6291	27	0.7756	—0.3916	43	0.1972	—0.7448
14	1.0771	0.5395	28	0.7440	—0.4277	44	0.1580	—0.7521
12	1.0765	0.4552	29	0.7116	—0.4617	45	0.1186	—0.7578
13	1.0722	0.3757	30	0.6783	—0.4938	46	0.07914	—0.7619
14	1.0646	0.3005	31	0.6443	—0.5939	47	0.03958	—0.7643
15	1.0540	0.2292	32	0.6096	—0.5520	48	0	—0.7651
16	1.0407	0.1617						

The problem is solved by the Fourier method. The expression for τ has the form

$$\nu_1\tau(\xi,\eta)=\frac{1}{\pi\rho}\int_0^\infty\frac{\cos p\xi\sin p\rho}{p}\times$$
$$\times\frac{A_1(p,R)I_0(p\eta)+A_2(p,R)K_0(p\eta)}{A_3(p,R)}dp,$$
$$A_1(p,R)=pK_1\left(\frac{pR}{R-1}\right)-B_2K_0\left(\frac{pR}{R-1}\right),$$
$$A_2(p,R)=pI_1\left(\frac{pR}{R-1}\right)+B_2I_0\left(\frac{pR}{R-1}\right),$$
$$A_3(p,R)=p^2\left[K_1\left(\frac{p}{R-1}\right)I_1\left(\frac{pR}{R-1}\right)-\right.$$
$$\left.-I_1\left(\frac{p}{R-1}\right)K_1\left(\frac{pR}{R-1}\right)\right]+$$
$$+pB_1\left[I_0\left(\frac{p}{R-1}\right)K_1\left(\frac{pR}{R-1}\right)+\right.$$
$$\left.+K_0\left(\frac{p}{R-1}\right)I_1\left(\frac{pR}{R-1}\right)\right]+$$
$$+pB_2\left[I_1\left(\frac{p}{R-1}\right)K_0\left(\frac{pR}{R-1}\right)+\right.$$
$$\left.+K_1\left(\frac{p}{R-1}\right)I_0\left(\frac{pR}{R-1}\right)\right]+$$
$$+B_1B_2\left[K_0\left(\frac{p}{R-1}\right)I_0\left(\frac{pR}{R-1}\right)-\right.$$
$$\left.-I_0\left(\frac{p}{R-1}\right)K_0\left(\frac{pR}{R-1}\right)\right]. \qquad (2.4)$$

Here I_0, I_1, K_0, and K_1 are Bessel functions of imaginary argument. Putting $\eta = 1/(R-1)+\eta'$, where η' is measured from the electrode surface heated by the arc, and letting $(R-1)$ tend to zero, we can show that (2.4) reduces to (1.10).

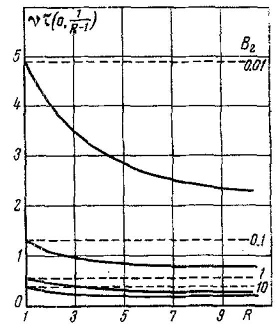

Fig. 4

Figure 4 shows the results of calculation of $\nu_1\tau\,(0, 1/(R-1))$ in the center of the spot for $B_1 = 0$ and $\rho = 1$ in relation to R and B_2. For comparison we give the values of $\nu\tau\,(0,0)$ in the plane approximation. The figure shows that $\nu_1\tau\,(0.1/(R-1))$ is less than $\nu\tau\,(0,0)$; in the range of R usually used in practice ($R \leqslant 2$) the difference does not exceed ~20%.

3. Plane electrode with annular spot. If the spot moves at high speed over the surface of a plane electrode (disk) in a closed annular path (Fig. 5) the spot will degenerate into a ring $2r_0$ wide, the central line of which is at a distance r_1 from the axis of rotation, along which the x-axis is directed; r is the distance from the given point to the axis of rotation. We consider an electrode of infinite extent. We introduce the dimensionless parameters

$$\xi=\frac{x}{\delta},\quad \eta=\frac{r}{\delta},\quad B=\frac{\alpha\delta}{\lambda},\quad \tau=\frac{\pi T'\lambda\delta}{Q},$$
$$\rho=\frac{r_0}{\delta},\quad R=\frac{r_1}{\delta},\quad \nu=2R.$$

The temperature T' is defined in the same way as in Section 1. The specific heat flux in the smeared spot is

$$q = Q\,/\,4\pi\, r_0 r_1. \qquad (3.1)$$

The heat-conduction equation has the form (2.1). The boundary conditions are

$$\frac{\partial\tau}{\partial\xi}=-\frac{1}{4\rho R}H\left[\rho^2-(R-\eta)^2\right]+B_1\tau,\qquad \xi=0;$$
$$\frac{\partial\tau}{\partial\xi}=-B_2\tau,\quad \xi=1;\qquad \frac{\partial\tau}{\partial\eta}=0,\quad \eta=0. \qquad (3.2)$$

The method of solution is the same as in Sections 1 and 2. As a result we obtain

$$\nu\tau(\xi,\eta)=\frac{1}{2\rho}\int_0^\infty J_0(p\eta)\Big\{(R+\rho)J_1[p(R+\rho)]-$$
$$-(R-\rho)J_1[p(R-\rho)]\Big\}\times$$
$$\times\frac{p\,\mathrm{ch}\,p(1-\xi)+B_2\,\mathrm{sh}\,p(1-\xi)}{(p^2+B_1B_2)\,\mathrm{sh}\,p+p(B_1+B_2)\,\mathrm{ch}\,p}dp. \qquad (3.3)$$

Here J_0 and J_1 are Bessel functions of real argument. Figure 6 shows the results of calculation of the distribution of $\nu\tau\,(0,\eta)$ over the electrode surface for different values of R with $B_1 = 0$, $B_2 = 1$, and $\rho = 0.01$. In the case $R = \rho$ the annular spot degenerates into a stationary spot with

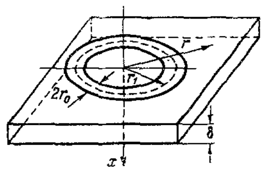

Fig. 5

radius $2r_0$. The temperature maximum in this case is exactly on the "edge" of the spot—on the axis of symmetry. With increase in R the maximum moves within the spot and when $R \geqslant 2\rho$ the maximum temperature does not differ greatly from the temperature on the central line of the spot.

4. Cylindrical electrode with spot moving at finite speed (plane approximation). We have given the solution to some problems of the temperature distribution in the electrode in the case where the arc spot moves at a "sufficiently high" speed over the electrode surface, i.e., for a "sufficiently high" frequency of passage of the spot through points lying in its path, so that the heat flux through the spot can be regarded as spread over the whole area of the region through which the spot passes. It is of interest to find out at what speed of the spot we can make this assumption and what is the difference between the actual temperature of an electrode with a moving spot and the temperature of an electrode with a smeared spot. We consider the plane approximation (Fig. 7). We imagine that the electrode is cut along a generatrix and opened out to form a plate. It is convenient to take a cut which is

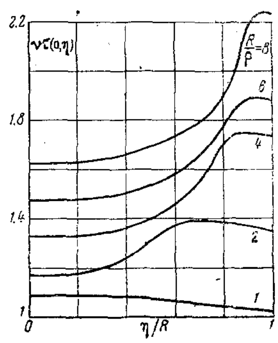

Fig. 6

fixed relative to the spot, i.e., which traverses the curved surface of the cylinder with the same speed V as the spot.

Taking the origin of coordinates at the center of the spot we find that the problem reduces to determination of the temperature field in a plate moving relative to the spot with the speed of its motion over the electrode, with boundary conditions on the surfaces of the cut, located at a fixed distance from the center of the spot, imposing the condition that each two corresponding points of these surfaces are actually one point on the initial cylinder. It is convenient to have the planes of the cut at equal distances from the center of the spot. The plate moves relative to the spot along the x-axis in a negative direc-

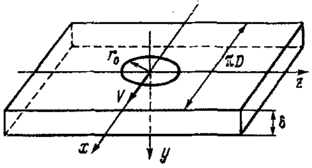

Fig. 7

tion. The specific heat flux in the arc spot is given by Eq. (0.1). The heat-conduction equation in the given case is the energy equation for a moving incompressible medium

$$\frac{\partial^2 T'}{\partial x^2}+\frac{V}{a}\frac{\partial T'}{\partial x}+\frac{\partial^2 T'}{\partial y^2}+\frac{\partial^2 T'}{\partial z^2}=0. \qquad (4.1)$$

The temperature T' is defined in the same way as in Section 1. We introduce the dimensionless parameters

$$\xi=\frac{x}{\delta},\quad \eta=\frac{y}{\delta},\quad \zeta=\frac{z}{\delta},\quad B=\frac{\alpha\delta}{\lambda},$$

$$\tau=\frac{\pi T'\lambda\delta}{Q},\quad \rho=\frac{r_0}{\delta},\quad \nu=\frac{D}{\delta},\quad \beta=\frac{V\delta}{a}.$$

The heat-conduction equation (4.1) takes the form

$$\partial^2\tau/\partial\xi^2+\beta\partial\tau/\partial\xi+\partial^2\tau/\partial\eta^2+\partial^2\tau/\partial\zeta^2=0. \qquad (4.2)$$

The boundary conditions on the surfaces y = 0, y = δ, and on the plane of symmetry z = 0 are written in the following way:

$$\frac{\partial\tau}{\partial\eta}=-\frac{1}{\rho^2}H(\rho^2-\xi^2-\zeta^2)+B_1\tau,\quad \eta=0;$$

$$\frac{\partial\tau}{\partial\eta}=-B_2\tau,\quad \eta=1;\qquad \frac{\partial\tau}{\partial\zeta}=0,\quad \zeta=0. \qquad (4.3)$$

The boundary conditions on the surfaces of the cut—the "periodicity conditions"—are

$$\tau\left(\xi=-\frac{\pi\nu}{2}\right)=\tau\left(\xi=\frac{\pi\nu}{2}\right),$$

$$\frac{\partial\tau}{\partial\xi}\left(\xi=-\frac{\pi\nu}{2}\right)=\frac{\partial\tau}{\partial\xi}\left(\xi=\frac{\pi\nu}{2}\right). \qquad (4.4)$$

The problem is solved by the Fourier method. We use the expansion

$$H(\rho^2-\xi^2-\zeta^2)=H(\rho-|\zeta|)H(\sqrt{\rho^2-\xi^2}-|\zeta|)=$$

$$=\frac{1}{\pi^2\nu}\sum_{n=-\infty}^{\infty}\exp\frac{2n\xi i}{\nu}\int_{-\infty}^{\infty}\frac{\cos p\zeta}{p}\times$$

$$\times\int_{-\rho}^{\rho}\exp\frac{2nti}{\nu}\sin p\sqrt{\rho^2-t^2}\,dt\,dp, \qquad (4.5)$$

and the equality [7]

$$\int_0^{\rho}\sin p\sqrt{\rho^2-t^2}\cos\frac{2nt}{\nu}\,dt=$$

$$=\frac{\pi p\rho}{2\sqrt{p^2+(2n/d)^2}}J_1[\rho\sqrt{p^2+(2n/\nu)^2}]. \qquad (4.6)$$

As a result we find

$$\nu\tau(\xi,\eta,\zeta)=\nu\tau_\infty(\eta,\zeta)+\psi(\xi,\eta,\zeta), \qquad (4.7)$$

$$\nu\tau_\infty(\eta,\zeta)=\frac{2}{\pi\rho}\int_0^\infty\frac{\cos p\zeta J_1(p\rho)}{p}\times$$

$$\times\frac{p\,\mathrm{ch}\,p(1-\eta)+B_2\,\mathrm{sh}\,p(1-\eta)}{(p^2+B_1B_2)\,\mathrm{sh}\,p+p(B_1+B_2)\,\mathrm{ch}\,p}\,dp, \qquad (4.8)$$

$$\psi(\xi,\eta,\zeta)=\frac{4}{\pi\rho}\sum_{n=1}^{\infty}\int_0^\infty\frac{\cos p\zeta J_1[\rho\sqrt{p^2+(2n/\nu)^2}]}{\sqrt{p^2+(2n/\nu)^2}\,(\theta_1+\theta_2)^2}\times$$

$$\times\left\{[\varphi_1(\eta)\theta_1+\varphi_2(\eta)\theta_2]\cos\frac{2n\xi}{\nu}+\right.$$

$$\left.+[\varphi_2(\eta)\theta_1-\varphi_1(\eta)\theta_2]\sin\frac{2n\xi}{\nu}\right\}dp, \qquad (4.9)$$

$$\varphi_1(\eta)=u\,\mathrm{ch}\,u(1-\eta)\cos v(1-\eta)-$$

$$-v\,\mathrm{sh}\,u(1-\eta)\sin v(1-\eta)+$$

$$+B_2\,\mathrm{sh}\,u(1-\eta)\cos v(1-\eta),$$

$$\varphi_2(\eta)=v\,\mathrm{ch}\,u(1-\eta)\cos v(1-\eta)+$$

$$+u\,\mathrm{sh}\,u(1-\eta)\sin v(1-\eta)+$$

$$+B_2\,\mathrm{ch}\,u(1-\eta)\sin v(1-\eta), \qquad (4.10)$$

$$\theta_1=(u^2-v^2+B_1B_2)\,\mathrm{sh}\,u\cos v-2uv\,\mathrm{ch}\,u\sin v+$$

$$+(B_1+B_2)\times(u\,\mathrm{ch}\,u\cos v-v\,\mathrm{sh}\,u\sin v),$$

$$\theta_2=(u^2-v^2+B_1B_2)\,\mathrm{ch}\,u\sin v+2uv\,\mathrm{sh}\,u\cos v+$$

$$+(B_1+B_2)(u\,\mathrm{sh}\,u\sin v+v\,\mathrm{ch}\,u\cos v), \qquad (4.11)$$

$$u={}^1/_2\sqrt{2}\,\{\{[p^2+(2n/\nu)^2]^2+$$

$$+\beta^2(2n/\nu)^2\}^{1/2}+p^2+(2n/\nu)^2\}^{1/2},$$

$$v={}^1/_2\sqrt{2}\,\{\{[p^2+(2n/\nu)^2]^2+$$

$$+\beta^2(2n/\nu)^2\}^{1/2}-p^2-(2n/\nu^2)^2\}^{1/2}. \qquad (4.12)$$

Function $\psi(\xi,\eta,\zeta)$ when $\beta\to\infty$ tends to zero, i.e., when the spot moves at high speed the electrode temperature is determined by the first term in (4.7). The difference from the corresponding expression in Section 1 consists in replacement of sin $p\rho$ by $2J_1(p\rho)$, which is due to variation of the specific heat flux over the width of the strip into which the round spot degenerates with constant distribution of the specific heat flux.

In this case

$$q(z)=4\pi^{-1}q_0\sqrt{1-(z/r_0)^2}. \qquad (4.13)$$

The value of q_0 is given by Eq. (0.1). Calculations show that the value of $\nu\tau_\infty(0,0)$ differs little from $\nu\tau(0,0)$ in Section 1.

At large values of β (even at moderate speeds of the spot the parameter β is fairly large) the expression for ψ becomes much simpler. It can be shown that on the surface $\eta = 0$ in this case

$$\psi(\xi,0,\zeta)\approx\psi_1(\xi,0,\zeta)=$$

$$=\frac{2\sqrt{\nu}}{\pi\rho\sqrt{\beta}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\left(\cos\frac{2n\xi}{\nu}-\sin\frac{2n\xi}{\nu}\right)\times$$

$$\times\int_0^\infty\frac{\cos p\zeta J_1[\rho\sqrt{p^2+(2n/\nu)^2}]\,dp}{\sqrt{p^2+(2n/\nu)^2}}. \qquad (4.14)$$

Here the integral is equal to zero outside the strip $|\zeta|\leq\rho$.

Hence, at large β the difference between the temperature of the electrode with a moving spot and the temperature produced by a smeared spot is concentrated mainly in the region through which the spot passes. Substituting the value of the integral, we find that in the strip

$$\psi_1(\xi,\,0,\,\zeta)=$$
$$=\frac{\nu^{3/2}}{2\pi\rho^2\sqrt{\beta}}\left\{a\left[\frac{2}{\nu}\left(\sqrt{\rho^2-\zeta^2}-\xi\right);\ \frac{3}{2}\right]+\right.$$
$$+a\left[\frac{2}{\nu}\left(\sqrt{\rho^2-\zeta^2}+\xi\right);\ \frac{3}{2}\right]-$$
$$-b\left[\frac{2}{\nu}\left(\sqrt{\rho^2-\zeta^2}-\xi\right);\ \frac{3}{2}\right]+$$
$$\left.+b\left[\frac{2}{\nu}\left(\sqrt{\rho^2-\zeta^2}+\xi\right);\ \frac{3}{2}\right]\right\},$$
$$a(x,\,s)=\sum_{n=1}^{\infty}\frac{\sin nx}{n^s},\qquad b(x,\,s)=\sum_{n=1}^{\infty}\frac{\cos nx}{n^s}. \qquad (4.15)$$

Table 1 gives values of $a = a(x, {}^3/_2)$, $b = b(x, {}^3/_2)$ for values of $X = 48\,x/\pi$, i.e., for values of x which are multiples of $\pi/48$ in the range $0 \leq x \leq \pi$ (functions $a(x, s)$ and $b(x, s)$ are periodic with a period of 2π).

We note that $\psi_1(\xi, 0, \zeta)$ is independent of the rate of cooling and supply of heat by the working medium. Thus, a reduction in the electrode temperature in excess of that due to the smeared spot can be effected only by an increase in the speed of the spot. Equation (4.15) takes a simpler form at large values of ν, when the sum can be replaced by the integral (β in this case must be $\gg \nu$, since otherwise (4.14) would not hold). Having in mind that the values of $2n\nu$ for successive values of n differ from one another by an increment $2/\nu$, which is small at large ν, we can denote the quantity $2n/\nu$ as a variable of integration. Then (4.15) can be written in the approximate form

$$\psi_1(\xi,\,0,\,\zeta)\approx\psi_1^*(\xi,\,0,\,\zeta)=$$
$$=\frac{\sqrt{2}\,\nu}{\pi\rho^2\sqrt{\beta}}\int_0^{\infty}\frac{\cos t\xi-\sin t\xi}{t^{3/2}}\sin t\sqrt{\rho^2-\zeta^2}\,dt\,. \qquad (4.16)$$

The integral in (4.16) is zero in the region in front of the spot, i.e., at large values of β and ν the excess temperature is concentrated only in the spot and its track. It can be shown that within the spot

$$\psi_1^*(\xi,\,0,\,\zeta)=\frac{2\nu}{\sqrt{\pi}\,\rho^2\sqrt{\beta}}\left(\sqrt{\rho^2-\zeta^2}-\xi\right)^{1/2}, \qquad (4.17)$$

and in the track ($\xi < -[\rho^2 - \zeta^2]^{1/2}$)

$$\psi_1^*(\xi,\,0,\,\zeta)=\frac{2\nu}{\sqrt{\pi}\,\rho^2\sqrt{\beta}}\left[\left(\sqrt{\rho^2-\zeta^2}-\xi\right)^{1/2}-\right.$$
$$\left.-\left(-\xi-\sqrt{\rho^2-\zeta^2}\right)^{1/2}\right]. \qquad (4.18)$$

The maximum value of ψ_1^* occurs precisely on the rear edge of the spot on the axis of motion $\xi = -\rho$, $\zeta = 0$ and is

$$\psi_{1\,\max}^*=\frac{2\sqrt{2}\,\nu}{\sqrt{\pi}\,\rho^{3/2}\sqrt{\beta}}, \qquad (4.19)$$

i.e., the maximum temperature increment in the moving spot is

$$\Delta T=\frac{2\sqrt{2}}{\pi\sqrt{\pi}}\,\frac{Q}{\lambda r_0^{3/2}}\left(\sqrt{\frac{a}{V}}\right)^{1/2}.$$

The form of function $\psi_1^*(\xi, 0, \zeta)$ is shown in Fig. 8.

We find the values of ν for which the approximate relationships (4.17) and (4.18) can be used. Strictly speaking, $\psi_1(\xi, 0, \zeta) = \psi_1^*(\xi, 0, \zeta)$ only when ν is infinitely large. If ν is finite, then in front of the spot $\psi \neq 0$. We assume that the value of ψ in front of the spot can be neglected if it comprises ε of the maximum value of ψ. Then the absolute value of ξ, where on the axis of motion ψ_1^* is equal to ε of $\psi_{1\max}^*$, will be approximately equal to the minimum value of $(\pi\nu - \rho)$ at which (4.17) and (4.18) can be regarded as valid.

This value $\nu_{\min}$ is

$$\frac{\nu_{\min}}{\rho}=\frac{1}{2\pi}\,\frac{(1+\varepsilon^2)^2}{\varepsilon^2}\approx\frac{1}{2\pi\varepsilon^2}\,. \qquad (4.20)$$

For an approximate estimate of the value of β at which ψ can be evaluated by means of ψ_1, we consider the second term ψ_2 of the ex-

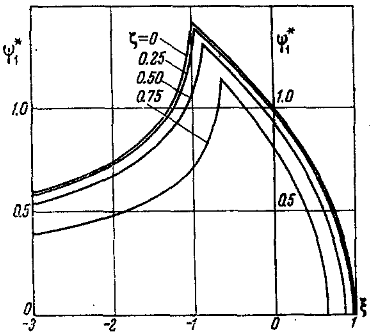

Fig. 8

pansion of ψ in terms of the small parameter $1/(\beta)^{1/2}$, and we find the values of β at which it can be neglected. We obtain

$$\psi_2(\xi,\,0,\,\zeta)=\frac{B_1\nu^2}{\pi\rho^2\beta}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{2n\xi}{\nu}\sin\left(\frac{2n}{\nu}\sqrt{\rho^2-\zeta^2}\right); \qquad (4.21)$$

in front of the spot ($\sqrt{\rho^2-\zeta^2}<\xi$)

$$\psi_2(\xi,\,0,\,\zeta)=\frac{B_1\nu}{\rho^2\beta}\sqrt{\rho^2-\zeta^2}\left(1-\frac{2\xi}{\pi\nu}\right); \qquad (4.22)$$

inside the spot ($|\xi|\leqslant\sqrt{\rho^2-\zeta^2}$)

$$\psi_2(\xi,\,0,\,\zeta)=\frac{B_1\nu}{\rho^2\beta}\,\xi\left(1-\frac{2}{\pi\nu}\sqrt{\rho^2-\zeta^2}\right); \qquad (4.23)$$

in the track ($\xi<-\sqrt{\rho^2-\zeta^2}$)

$$\psi_2(\xi,\,0,\,\zeta)=-\frac{B_1\nu}{\rho^2\beta}\sqrt{\rho^2-\zeta^2}\left(1+\frac{2\xi}{\pi\nu}\right). \qquad (4.24)$$

The greatest value of ψ_2 occurs on the front edge of the spot and is

$$\psi_{2\,\max}=\frac{B_1\nu}{\rho\beta}\left(1-\frac{2\rho}{\pi\nu}\right). \qquad (4.25)$$

Hence, ψ_2 will be much less than ψ_1 if (in view of (4.19) and assuming $(2\rho/\pi\nu) \ll 1$)

$$\beta\gg{}^1/_8\pi\rho\,B_1^2\,. \qquad (4.26)$$

It is clear that ψ_2, like ψ_1, is independent of the cooling rate. If the convective heat flux to the electrode surface in contact with the working medium is small, i.e., $B_1 \approx 0$, then to determine the lower limit of the values of β at which ψ can be evaluated by means of ψ_1, we must investigate the third term ψ_3 of the expansion of ψ in terms of the parameter $1/(\beta)^{1/2}$, which depends on B_2. Assuming $B_1 = 0$, we can evaluate ψ_3 from the expression (for large ν)

$$\psi_3<\frac{5.2}{\pi}\,\frac{B_2^2\,\nu^{3/2}}{\rho\beta^{3/2}}\,. \qquad (4.27)$$

Hence, it follows that ψ_1 will be much less than ψ_3 if

$$\beta\gg(2.6\,/\,\sqrt{2\pi})\,B_2^2\,\sqrt{\nu\rho}\,. \qquad (4.28)$$

The value of ψ on the electrode surface in contact with the cooling medium for large β can be put in the form

$$\psi(\xi,\,1,\,\zeta)\approx\psi_1(\xi,\,1,\,\zeta)=$$
$$=\frac{8}{\pi\rho}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\cos p\zeta J_1\{\rho[p^2+(2n/\nu)^2]\}\,e^{-u}}{\sqrt{p^2+(2n/\nu)^2}\,(u^2+v^2)}\times$$
$$\times\left[u\cos\left(v+\frac{2n\xi}{\nu}\right)-v\sin\left(v+\frac{2n\xi}{\nu}\right)\right]dp\,. \qquad (4.29)$$

The value of $\psi_1(\xi, 1, \zeta)$ is independent of B_1 and B_2, i.e., the major part of the temperature in excess of that produced by a smeared spot can be reduced only by increasing the speed of the spot. The estimate (4.28) shows that

$$\psi_1(\xi, 1, \zeta) < \Psi_1 = \frac{2\sqrt{2}}{\pi} \frac{\nu}{\rho^2} \left(\frac{\nu}{\beta}\right)^{1/2} \exp\left[-\left(\frac{\beta}{\nu}\right)^{1/2}\right]. \quad (4.30)$$

In view of the exponential relationship between ψ_1 and $-(\beta/\nu)^{1/2}$ this value is usually small in comparison with $\nu\tau_\infty(\xi, 1, \zeta)$.

REFERENCES

1. N. N. Rykalin, Thermal Principles of Welding [in Russian], Izd-vo AN SSSR, 1947.

2. W. Lai and R. V. Dunkle, "Diffusion of heat through a disk source," Teploperedacha, Translated from Trans. ASME, no. 3, p. 85, 1962.

3. V. I. Krylovich, "Velocity of arc in an electric-arc heater," Inzh.-fiz. zh., no. 1, p. 114, 1963.

4. V. I. Krylovich, "Temperature field in the cooled electrode of an electric-arc heater with a stationary arc," Inzh.-fiz. zh., no. 3, p. 81, 1963.

5. V. I. Krylovich, "Temperature field in an electric-arc heater with a rotating arc," Inzh.-fiz. zh., no. 8, p. 70, 1963.

6. V. I. Smirnov, Course of Higher Mathematics [in Russian], vol. 2, Fizmatgiz, 1958.

7. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], Fizmatgiz, 1962.

27 June 1966 Novosibirsk